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Being Tactical About Trading & Taxes

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Abstract
In this paper we address the issue of balancing the value of being tactical with the implications of short versus long-term capital gains tax rates. As active managers, we believe our models can provide information about the future distribution of returns; however, a trade-off decision should be made about whether to act immediately upon a signal and suffer short-term capital gains tax rates or defer action until after our returns are long-term qualified ("tax maturity").

Introduction

Tactical strategies are largely recognized as being tax-inefficient. In a prior paper, *Being Strategic about Tactical Allocations*, we explored reducing overall portfolio turnover – which will oftentimes increase tax efficiency – through optimization methods that balance turnover costs with tracking error.[1] This method, however, was agnostic to tax rates and the role of holding time, which are large drivers of tax-based investment decisions.

While a purely passive portfolio would be the most tax-efficient implementation, if we believe that our models provide insight into future return distributions, then we should compare the trade off of acting on a signal immediately and suffering short-term capital gains tax rates versus deferring action until after our gains are long-term qualified ("tax maturity").

To Act or to Defer?

Let us denote our long-term and short-term capital gains tax rates as *LT* and *ST*, respectively. Also let us assume that we purchased a stock at price X_0 and currently hold it at time *t* with price X_t, where $X_t > X_0$.

Consider receiving a signal that dictates selling the security. We can explore the relationship of profitability from selling now, paying the short-term capital gains taxes, and reinvesting into a risk-free instrument (at the risk-free rate r) versus waiting until time *T* when we are eligible for long-term capital gains treatment.[2] Specifically, it is beneficial to wait when the expected value of the spread between these two options is negative:

$$E\left[(X_t - X_0)\times(1 - ST)\times e^{r(T-t)} - (X_T - X_0)\times(1 - LT) \;\middle|\; F(t)\right] < 0$$

[1] http://cdn.thinknewfound.com/wp-content/uploads/2011/05/Being-Strategic-About-Tactical-Allocations.pdf

[2] We are assuming that the short term capital gains tax is paid immediately when the security is sold for simplicity since the time when taxes are actually paid will not coincide with the tax maturity, in general. This can be thought of as holding the short-term capital gains tax payment in a zero interest account.

Collecting terms and rewriting,

$$(X_t - X_0)\times\frac{e^{r(T-t)}(1-ST)}{1-LT} + X_0 < E[X_T \mid F(t)]$$

If we assume we can model stock price, *X*, as Geometric Brownian Motion, then we can model X_T as:

$$X_T = X_t e^{\left(\mu-\frac{\sigma^2}{2}\right)\times(T-t)+\sigma\sqrt{(T-t)}Z}$$

Where Z is a standard normal. Therefore,

$$\frac{(X_t - X_0)\times\frac{e^{r(T-t)}(1-ST)}{1-LT} + X_0}{X_t} < E[e^{\left(\mu-\frac{\sigma^2}{2}\right)\times(T-t)+\sigma\sqrt{(T-t)}Z}]$$

where, if we define our current “percent gain” as p:

$$\frac{X_0}{X_t} = \left(\frac{1}{1+p}\right)$$

and therefore,

$$\frac{X_t - X_0}{X_t} = p\times\left(\frac{1}{1+p}\right)$$

We define:

$$\mathrm{k} = \left(\frac{1}{1+p}\right)\times\left(p\times\frac{e^{r(T-t)}(1-ST)}{1-LT} + 1\right)$$

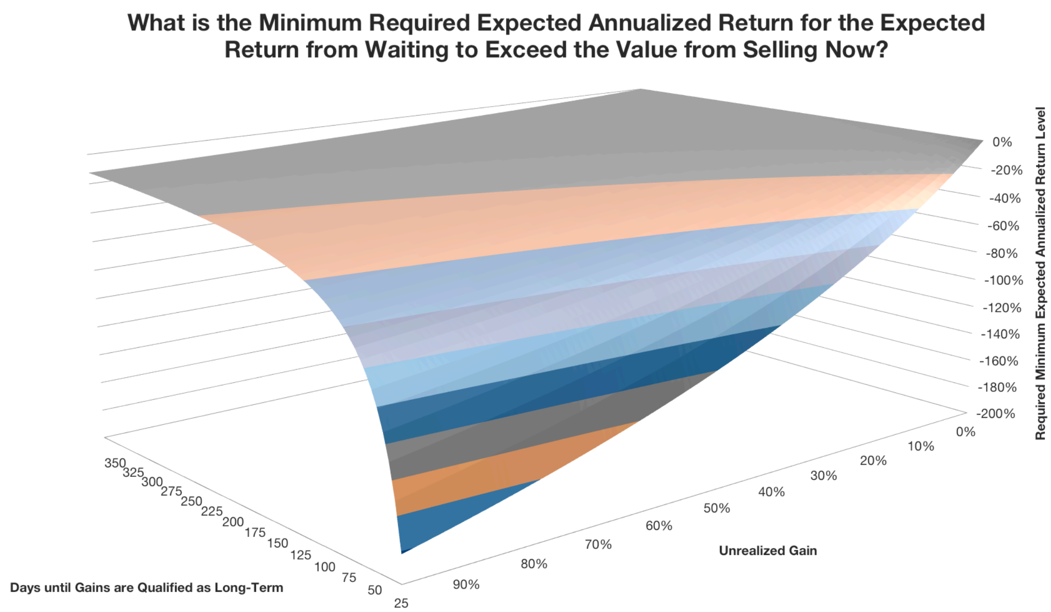
Since we know the quantity within the expected value is log-normally distributed, we find that the expected value is $e^{\mu(T-t)}$. Therefore, the μ that satisfies the first inequality is:

$$\mu > \frac{\log(\mathrm{k})}{(T-t)}$$

What we see is that our relationship relies on:

- Short-term capital gains and long-term capital gains tax rates
- Our current return level
- How long until we are eligible for long-term capital gains treatment

Perhaps most interestingly, we see that the volatility component completely falls out of the equation, a point we will return to later. Plotting the surface μ (assuming a risk-free rate of 0%) has an intuitive interpretation: the closer we are to tax maturity and the larger our unrealized gain, the more negative the annualized expected return will have to be to justify taking immediate action. We have assumed that the long-term tax rate is 15% and the short-term tax rate is 35%.



The table below provides rough minimum expected annualized return values required to justify holding onto the security based on the current unrealized gain level and days until tax maturity.

	5	50	100	150	200	250	300	350
0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
5%	-82.25%	-8.23%	-4.11%	-2.74%	-2.06%	-1.65%	-1.37%	-1.18%
10%	-157.84%	-15.78%	-7.89%	-5.26%	-3.95%	-3.16%	-2.63%	-2.25%
15%	-227.55%	-22.76%	-11.38%	-7.59%	-5.69%	-4.55%	-3.79%	-3.25%
20%	-292.04%	-29.20%	-14.60%	-9.73%	-7.30%	-5.84%	-4.87%	-4.17%
25%	-351.88%	-35.19%	-17.59%	-11.73%	-8.80%	-7.04%	-5.86%	-5.03%
30%	-407.55%	-40.75%	-20.38%	-13.58%	-10.19%	-8.15%	-6.79%	-5.82%
35%	-459.48%	-45.95%	-22.97%	-15.32%	-11.49%	-9.19%	-7.66%	-6.56%
40%	-508.03%	-50.80%	-25.40%	-16.93%	-12.70%	-10.16%	-8.47%	-7.26%
45%	-553.53%	-55.35%	-27.68%	-18.45%	-13.84%	-11.07%	-9.23%	-7.91%
50%	-596.25%	-59.62%	-29.81%	-19.87%	-14.91%	-11.92%	-9.94%	-8.52%
60%	-674.33%	-67.43%	-33.72%	-22.48%	-16.86%	-13.49%	-11.24%	-9.63%
65%	-710.09%	-71.01%	-35.50%	-23.67%	-17.75%	-14.20%	-11.83%	-10.14%
70%	-743.92%	-74.39%	-37.20%	-24.80%	-18.60%	-14.88%	-12.40%	-10.63%
75%	-775.95%	-77.60%	-38.80%	-25.87%	-19.40%	-15.52%	-12.93%	-11.09%
80%	-806.34%	-80.63%	-40.32%	-26.88%	-20.16%	-16.13%	-13.44%	-11.52%
85%	-835.20%	-83.52%	-41.76%	-27.84%	-20.88%	-16.70%	-13.92%	-11.93%
90%	-862.64%	-86.26%	-43.13%	-28.75%	-21.57%	-17.25%	-14.38%	-12.32%
95%	-888.78%	-88.88%	-44.44%	-29.63%	-22.22%	-17.78%	-14.81%	-12.70%

For instance, if our unrealized gain was 65% and we were 200 days from tax maturity, we would have to believe that the expected annualized return rate would be less than -17.75% for the value of selling now to be greater than the expected value of waiting until tax maturity.

One in the Hand

The above analysis presumes we care about what happens *on average*. Optimizing decisions to the average is relevant to when it is possible to make numerous independent and identical bets. For example, if we held 30 securities in our portfolio whose trading decisions were completely independent, our above analysis would be relevant. However, assuming that trading signals between securities will be independent is likely unrealistic since security prices often share the same underlying risk factors that drive performance. Another possibility is if we could make these decisions independently throughout time; unfortunately, here we are limited by the length of time it takes for an investment to reach tax maturity.

Since it cannot be easily or quickly "averaged away," uncertainty will play a large role in realized results. In this case, *one bird in the hand* may be better than *two* *in the bush* (in the case of current tax rates, it's really more like 1.307 in the bush). Worst-case scenarios may then be worth consideration because we can quantify our downside. To do this, we can re-approach the problem and say that we will only defer action if $(X_T - X_0)\times(1 - LT) > (X_t - X_0)\times(1 - ST)\times e^{r(T-t)}$ with some degree of certainty c:

$$\mathbb{P}\big((X_T - X_0)\times(1 - LT) > (X_t - X_0)\times(e^{r(T-t)} - (1 - ST))\big) = c$$

By solving in a similar way as before, we find that:

$$\mathbb{P}\left(Z \le \frac{log(k) - \left(\mu - \frac{\sigma^2}{2}\right)\times(T-t)}{\sigma\sqrt{(T-t)}}\right) = 1 - c$$

Unfortunately, μ is not easily extracted as it now relies on p, $(T-t)$, σ and c. However, it can be found numerically for given values of the other parameters.

Valuing Opportunities

Our above analysis assumes that we are comparing selling now (and holding our profit in a risk-free return vehicle) versus holding onto the security and selling at tax maturity. A more realistic assumption is that we are going to reinvest our capital into *another* security. We will represent the second investment again by Geometric Brownian Motion:

$$\big((X_t - X_0)\times(1 - ST) + X_0\big)e^{\left(\mu_2 - \frac{{\sigma_2}^2}{2}\right)\times(T-t) + \sigma_2\sqrt{(T-t)}Z_2} - \big((X_T - X_0)\times(1 - LT) + X_0\big)$$

where Z_2 is another standard normal with a correlation of ρ to Z_1.[3] Unfortunately, disentangling the difference to find an analytical relationship between the two expected annualized returns is impossible to derive because there is no closed-form solution to the sum of two lognormal variables. However, if we re-write the final profit spread as:

[3] This formula assumes that asset 1 is sold at tax maturity and that the taxes owed are paid upon sale. Of course, at this time, it may still be advantageous to continue holding the asset. It also takes no allowance for accrued tax gains or losses from asset 2.

$$X_t\left(1-\frac{p\times ST}{1+p}\right)e^{\left(\mu_2-\frac{\sigma_2{}^2}{2}\right)\times(T-t)+\sigma_2\sqrt{(T-t)}Z_2}-X_t(1-LT)e^{\left(\mu_1-\frac{\sigma_1{}^2}{2}\right)\times(T-t)+\sigma_1\sqrt{(T-t)}Z_1}-X_t\left(\frac{LT}{1+p}\right)$$

we can establish the following definition of our resulting profit spread, *S*:

$$S = Ae^{r_2} - Be^{r_1} - K$$

Where:

$$A = X_t\left(1-\frac{p\times ST}{1+p}\right) \qquad B = X_t(1-LT)$$

$$K = X_t\left(\frac{LT}{1+p}\right)$$

$$r_1 = \left(\mu_1-\frac{\sigma_1{}^2}{2}\right)\times(T-t)+\sigma_1\sqrt{(T-t)}Z_1$$

$$r_2 = \left(\mu_2-\frac{\sigma_2{}^2}{2}\right)\times(T-t)+\sigma_2\sqrt{(T-t)}Z_2$$

If we take the probability of the spread being greater than 0, we get

$$\mathbb{P}(Ae^{r_2} - Be^{r_1} - K > 0)$$

which is equivalent to:

$$\mathbb{E}^{\mathbb{P}}[1_{Ae^{r_2}-Be^{r_1}>K}]$$

This form looks incredibly similar to the price of a digital spread option with a strike:

$$P = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\left[1_{S_T^{(2)}>S_T^{(1)}+K}\right]$$

with $S_T^{(2)} = Ae^{r_2}$, $S_T^{(1)} = Be^{r_1}$, and K as defined above. The probability we desire is found under the physical probability measure (rather than the risk-neutral measure), and we do not require discounting.

Unfortunately, such an option has no closed-form solution because the sum of lognormal random variables is not lognormal. There are, however, approximation methods.[4,5] Both methods can be easily adapted to calculate the probability of the spread being positive. The pricing formulas under the risk-neutral measure use the risk free rate as the drift for the assets; under the physical measure, we must substitute in the actual drift to find the real-world probability.

Whereas in the previous section we derived a formula for the probability of a positive spread (over the risk free asset) in terms of the expected annualized return on one security, now, we can approximate the probability of having a positive profit (price of the spread option) over a range of expected annualized returns for both securities. Using Li et. al. (2008), we can express the probability of the spread being positive as:

$$\mathbb{P}(S > 0) \approx J_0(C, D) + J_1(C, D)\varepsilon + \frac{1}{2} J_2(C, D)\varepsilon^2$$

where:

$$J_0(u, v) = N\left(\frac{u}{\sqrt{1 + v^2}}\right)$$

$$J_1(u, v) = \frac{1 + (1 + u^2)v^2}{(1 + v^2)^{\frac{5}{2}}} n\left(\frac{u}{\sqrt{1 + v^2}}\right)$$

$$J_2(u, v) = \frac{6(1 - u^2)v^2 + (21 - 2u^2 - u^4)v^4 + 4(3 + u^2)v^6 - 3}{(1 + v^2)^{\frac{11}{2}}} u \cdot n\left(\frac{u}{\sqrt{1 + v^2}}\right)$$

$$R = Be^{\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)(T - t)} \qquad \varepsilon = \frac{-RK\sigma_1^2\sqrt{T - t}}{2\sigma_2\sqrt{1 - \rho^2}(R + K)^2}$$

$$C = \frac{1}{\sigma_2\sqrt{T - t}\sqrt{1 - \rho^2}}\left(\log\left(\frac{A}{R + K}\right) + \left(\mu_2 - \frac{1}{2}\sigma_2^2\right)(T - t)\right)$$

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[4] See Li, Minqiang and Deng, Shijie and Zhou, Jieyun, Closed-Form Approximations for Spread Option Prices and Greeks (21 Jan 2008). Available at SSRN: http://ssrn.com/abstract=952747 or http://dx.doi.org/10.2139/ssrn.952747

[5] See Carol Alexander and Andrew Scourse. Bivariate normal mixture spread option valuation. Quantitative Finance, 4:637–648, 2004.

$$D = \frac{1}{\sigma_2\sqrt{1-\rho^2}}\left(\rho\sigma_2 - \frac{\sigma_1 R}{R+K}\right)$$

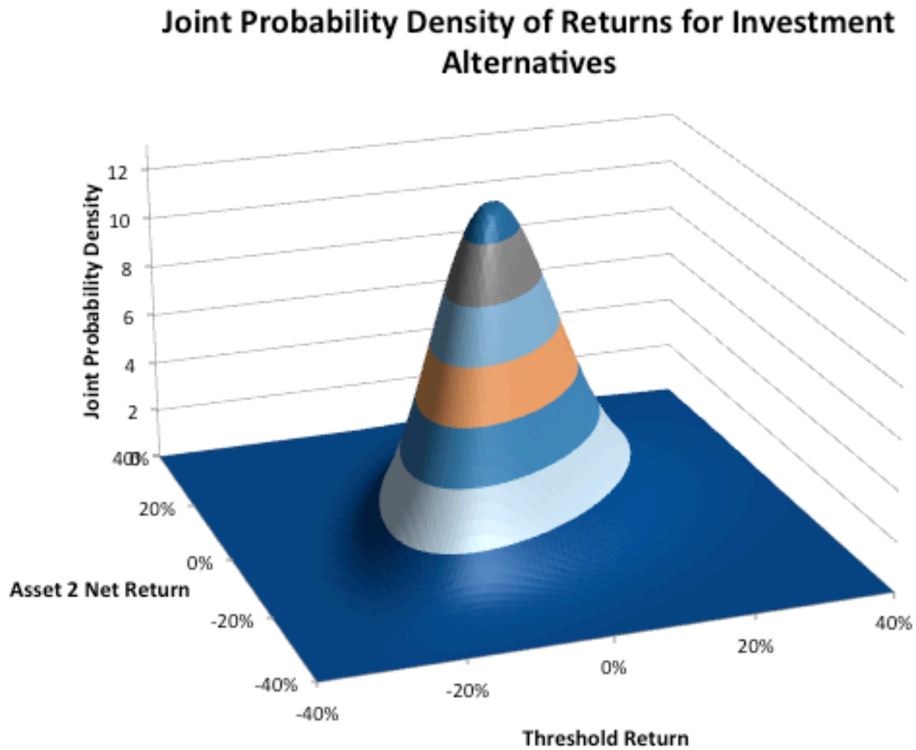
N(x) is the cumulative probability of the Gaussian distribution function; *n(x)* is the density of the Gaussian distribution function[6].

Distilling It Down Graphically

The probability is dependent on the individual security volatilities, expected returns, and correlation, along with the time remaining in the year and the current profit, *p*, which encapsulates the dependency on X_0 and X_t. Having so many knobs to turn makes full visualization difficult. However, we can gain some intuition by investigating how the joint probability density function of the assets shifts as the parameters vary.

The following plot shows the joint probability density function for the investment alternatives for given set of parameters.

[6] We are assuming that $K > 0$ to use this method, which is true since $p > -1$. However, if we had $K < 0$, we could simply switch the roles of asset 1 and 2.

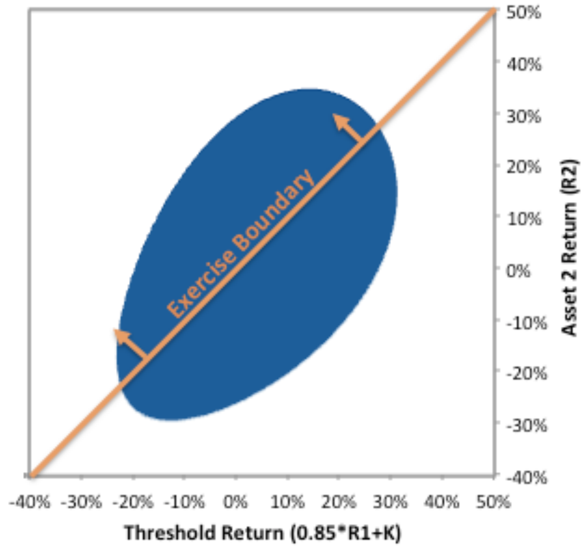


$$T = 0.75, \sigma_1 = 0.15, \sigma_2 = 0.15, \mu_1 = 0, \mu_2 = 0, \rho = 0.5, p = 0.1$$

Altering the parameters will change the location, size, and shape of the joint density function. However, we can possibly understand transformations to this object better in two dimensions rather than three. The following graphic shows the 95% contour of the joint probability density function over a range of returns for the assets. If we picture the blue "egg" as a bubble rising out of the page (from the previous plot), we can imagine slicing vertically through it at a 45° angle. This line marks the cutoff where the return on the after tax amount invested in asset 2 has surpassed the after tax return on asset 1 plus the required hurdle rate, *K*.[7] The volume of the bubble above this line is the probability we are after.

[7] The after tax return is at the short-term rate for asset 2 since we had to sell asset 1 to do this. The after tax return is at the long-term rate for asset 1.

Joint Probability of Returns for Investment Alternatives (Contour Plot)



$T = 0.75, \sigma_1 = 0.15, \sigma_2 = 0.15, \mu_1 = 0, \mu_2 = 0, \rho = 0.5, p = 0.1$
Probability = 0.433

The subsequent graphs show how changing the parameters changes the bubble (shifting, stretching, shrinking, tilting, etc.). By visualizing how the probability density will morph, we can gain insight into how likely we are to overcome the tax impact if we sell immediately and pay the short term capital gains tax, depending on our knowledge of the two assets. The light blue shows the original density, and the parameter changes are indicated.

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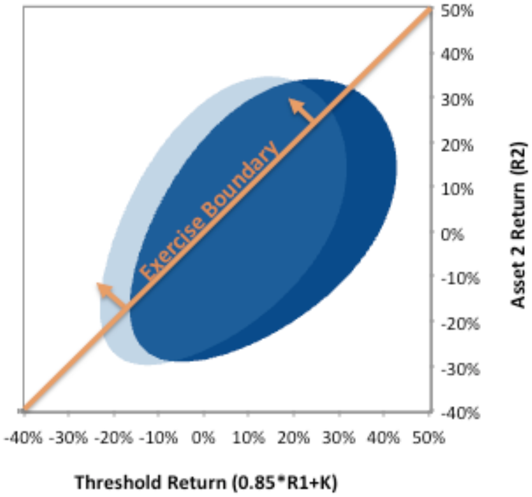
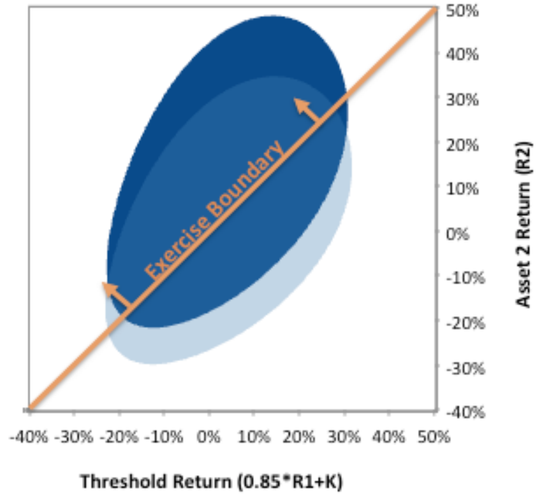
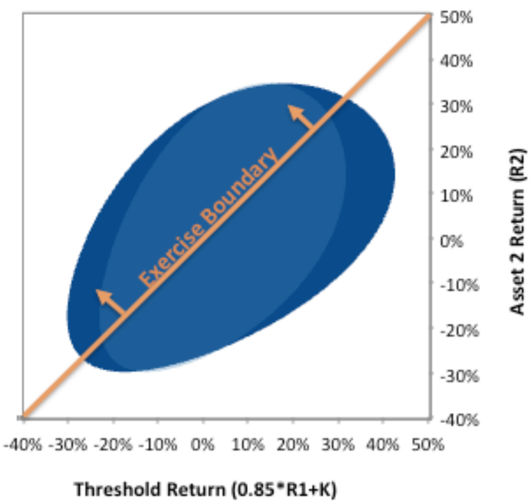
Joint Probability of Investment Alternatives with Altered Parameters



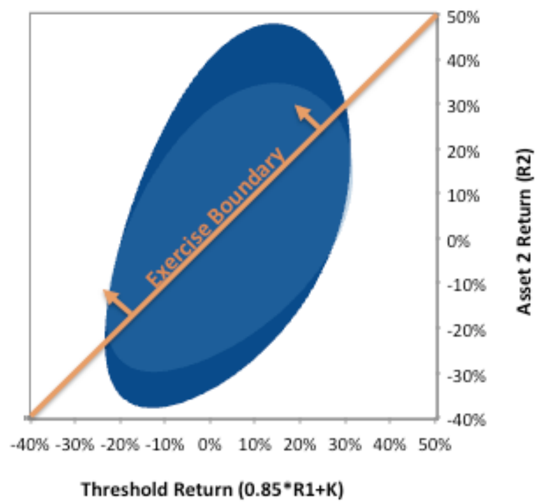
$\mu_1 = 0.1$
Probability = 0.242



$\mu_2 = 0.1$
Probability = 0.674

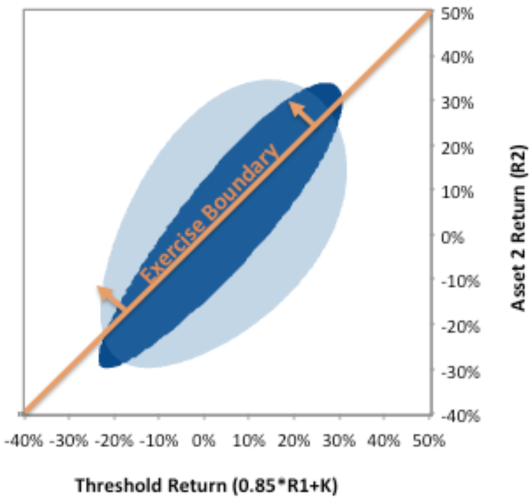


$\sigma_1 = 0.2$
Probability = 0.456

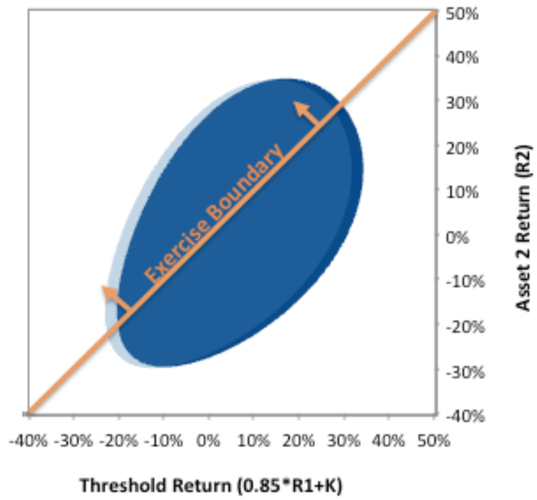


$\sigma_2 = 0.2$
Probability = 0.429

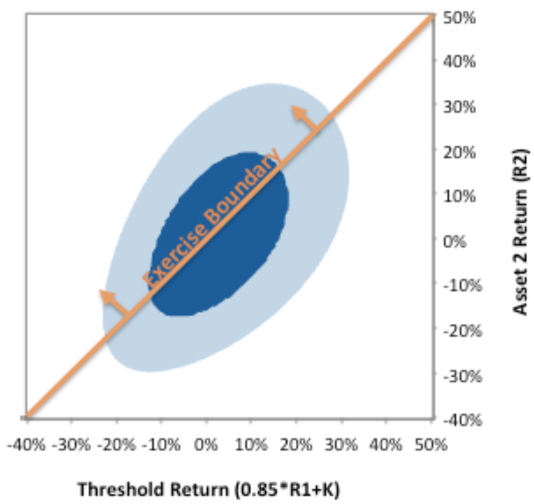
Joint Probability of Investment Alternatives with Altered Parameters



correlation, $\rho = 0.9$
Probability = 0.360



current profit, $p = 0.3$
Probability = 0.344



$T = 0.25$
Probability = 0.393

Distilling It Down Heuristically

Another way we can get a more general idea of the functional behavior of the probability is by examining the partial derivatives of the probability function (equivalent to analyzing the Greeks for options). The following heuristics hold for commonly encountered parameter sets:[8]

1. **Expected Return** – The probability of a positive spread increases with μ_2 and decreases with μ_1. This agrees with intuition since higher average returns for the asset we are buying compared to those for the asset we are selling should increase our chances of covering the loss from the higher, short-term tax rate. The effect from μ_2 is larger than that from μ_1 (absolute value).

2. **Current Profit** – Increasing the current profit, *p*, decreases the probability because a higher profit decreases the amount available to invest into asset 2. Thus, when deciding between selling two very similar assets, the one with the lowest current profit is generally the best option.

3. **Asset Volatility** – Generally, higher volatility for both assets increase the probability because they widen the final distribution of stock prices. However, there are some instances when it will decrease the probability. The extent of this effect depends on the correlation, but relative to the expected return expectations, the effect of varying volatility is small. Both effects are stronger for higher current profits and shorter time left until the end of the year.

4. **Time** – Selling sooner generally yields a higher probability of a net profit, especially for higher values of current profit, assuming that $\mu_2 > \mu_1$.

5. **Correlation** – As the correlation between the assets increases, the probability generally decreases since the assets behave more identically and cannot overcome the required spread. However, this is less pronounced with more time to tax maturity and lower current profits.

[8] These heuristics are based on parametric runs with ranges $\mu_i = [-0.25, 0.25]$, $\sigma_i = [0.05, 0.45]$, $p = [0.02, 0.5]$, $T = [0.025, 0.9]$, and $\rho = [-0.9, 0.9]$.

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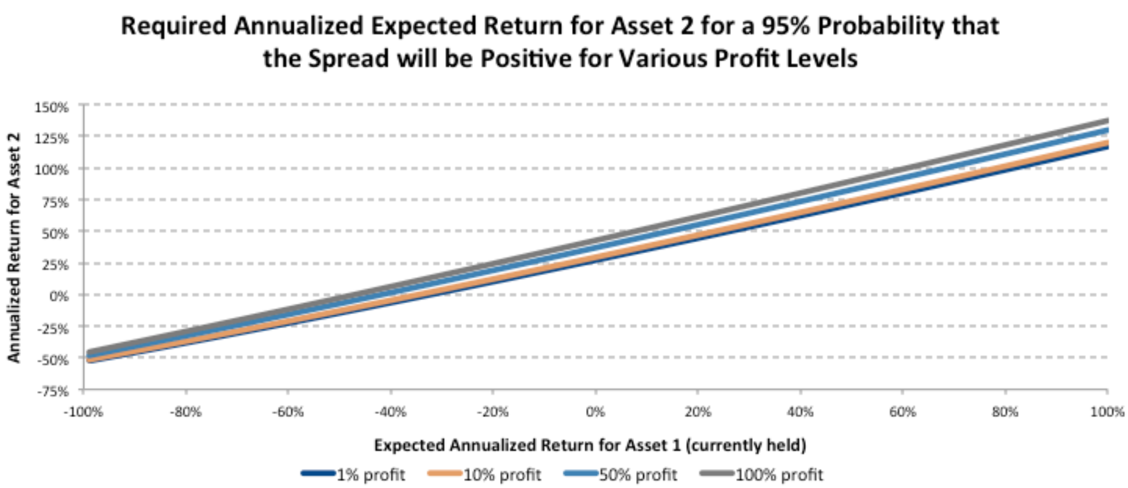
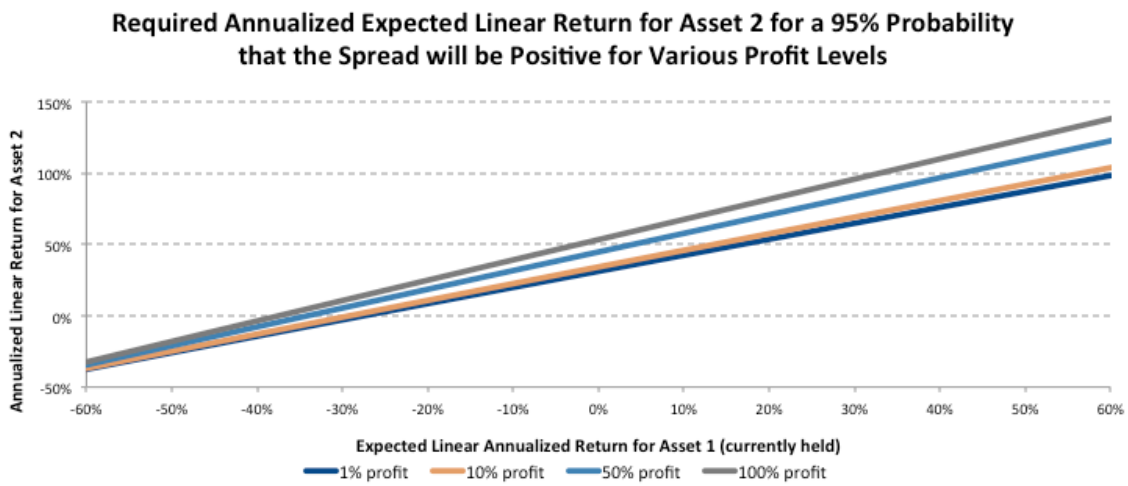
How Strong are Your Beliefs?

As we are focusing on individual trades rather than many independent bets, it is desirable to have a high probability of a positive spread rather than a positive expected value. We can solve for this probability level numerically using the method outlined earlier.

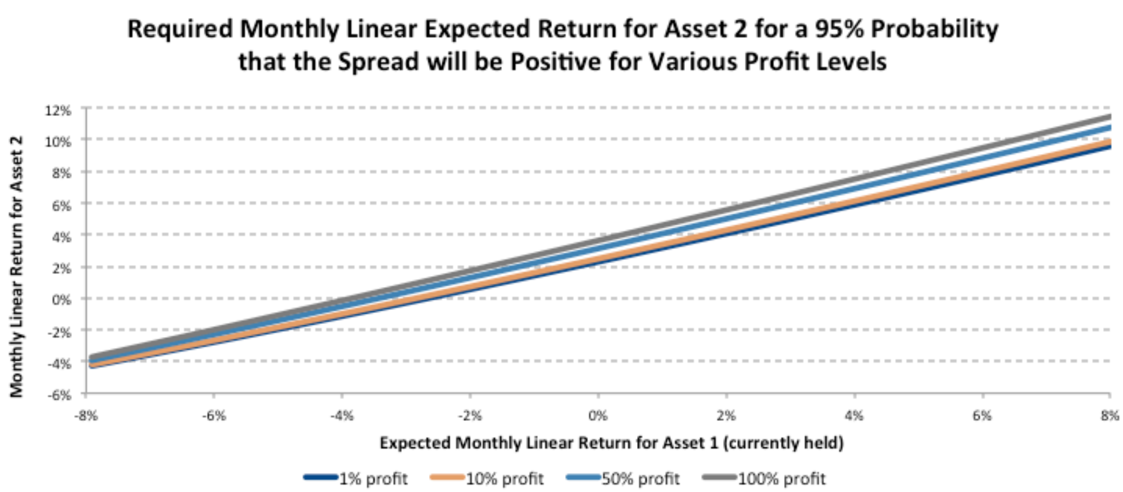
The following graph shows the expected return necessary to justify switching to asset 2 for a given expected return on asset 1 for various levels of current profit with $T = 0.75, \sigma_1 = 0.15, \sigma_2 = 0.15$, and $\rho = 0.5$.



However, we must be careful how we interpret this: the "expected annualized return" that is used in Geometric Brownian Motion is logarithmic return (i.e. continuously compounded), whereas most of our intuition relates to linear returns, which are what our investments actually realize.



Because all of these decisions are made on a time scale of less than a year, perhaps an even better illustration is the *monthly* returns on each asset, keeping in mind that these returns are for each month until tax maturity.



The approximation for the probability presented earlier was based on worked done for valuing spread options. The authors of the method claim that it is generally accurate to within 0.01%. For our purposes, we are not valuing options and can tolerate some inaccuracy: valuing an option at $0.88 when its true value is $0.92 has a very large impact when you are selling positions sized in the millions, but estimating the probability of a positive spread at 88% versus 92% might not change one's decision to make the trade.

If we neglect the terms involving epsilon in the approximation, then the probability is simply given by J_0, which is a cumulative normal density function.[9] Thus, if the argument to the function is greater than 1.65, there is more than a 95% probability that trade will be profitable.

Mathematically speaking, we are left with:

$$\mathbb{P}(S > 0) \approx N\left(\frac{C}{\sqrt{1 + D^2}}\right)$$

where we desire:

$$1.65 < \frac{C}{\sqrt{1 + D^2}} = \frac{\left(\mu_2 - \frac{1}{2}\sigma_2^2\right)(T - t) + \log(A) - \log\left(R + K\right)}{\sqrt{T - t}\sqrt{\sigma_2^2 - \frac{2\rho\sigma_1\sigma_2 R}{R + K} + \left(\frac{\sigma_1 R}{R + K}\right)^2}}$$

$$\mu_2 > \frac{1}{T - t}\left[1.65\sqrt{T - t}\sqrt{\sigma_2^2 - \frac{2\rho\sigma_1\sigma_2 R}{R + K} + \left(\frac{\sigma_1 R}{R + K}\right)^2} - \log\left(A\right) + \log(R + K)\right] + \frac{1}{2}\sigma_2^2$$

We are now to a point where we have an expression that gives a minimum expected return for asset 2 to achieve a 95% probability of a profitable spread.

We can treat the term with the square root that involves volatility as a volatility of the spread: higher correlation reduces this volatility while lower correlation increases the spread volatility. When we do this, it looks very similar to a standardized z-score, with $\log(R + K) - \log\left(A\right) - \frac{1}{2}\sigma_2^2(T - t)$ as the mean.

We can also simplify this further by linearizing it around $\mu_1 = 0$. For common choices of parameters, the slope of the line is between 0.85 and 0.90. If we know the minimum expected return on asset 2 when the expected return on asset 1 is 0, then we can quickly approximate the new expected return on asset 2 if we change our views of the expected return on asset 1.

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[9] Neglecting these terms leaves us with an upper bound for the probability, which, the authors state, is generally still within 0.1% of the true value for commonly encountered parameters.

The following tables show the slope and intercept for different parameter sets. These tables can be used to approximate the required return on asset 2 for a given belief of the return on asset 1.

$$\mu_2 > slope \cdot \mu_1 + intercept$$

The units are in percentage monthly return. For these parameters along with others (e.g. higher volatilities), a reasonably conservative estimate for the slope is 0.90.

Correlation	-90%	**Time (T-t)**	0.25			0.5			0.75		
		Profit, p	10%	20%	30%	10%	20%	30%	10%	20%	30%
10%	10%	**Intercept**	5.62%	6.18%	6.68%	3.84%	4.13%	4.39%	3.09%	3.29%	3.46%
		Slope	0.87	0.88	0.89	0.87	0.88	0.89	0.88	0.89	0.89
10%	20%	**Intercept**	8.45%	9.02%	9.51%	5.89%	6.17%	6.43%	4.78%	4.98%	5.15%
		Slope	0.87	0.88	0.89	0.87	0.88	0.89	0.88	0.89	0.89
20%	10%	**Intercept**	7.83%	8.42%	8.94%	5.38%	5.68%	5.95%	4.32%	4.53%	4.72%
		Slope	0.88	0.89	0.90	0.89	0.90	0.90	0.89	0.90	0.91
20%	20%	**Intercept**	10.62%	11.21%	11.73%	7.39%	7.69%	7.96%	5.99%	6.20%	6.38%
		Slope	0.88	0.89	0.90	0.89	0.90	0.90	0.89	0.90	0.91

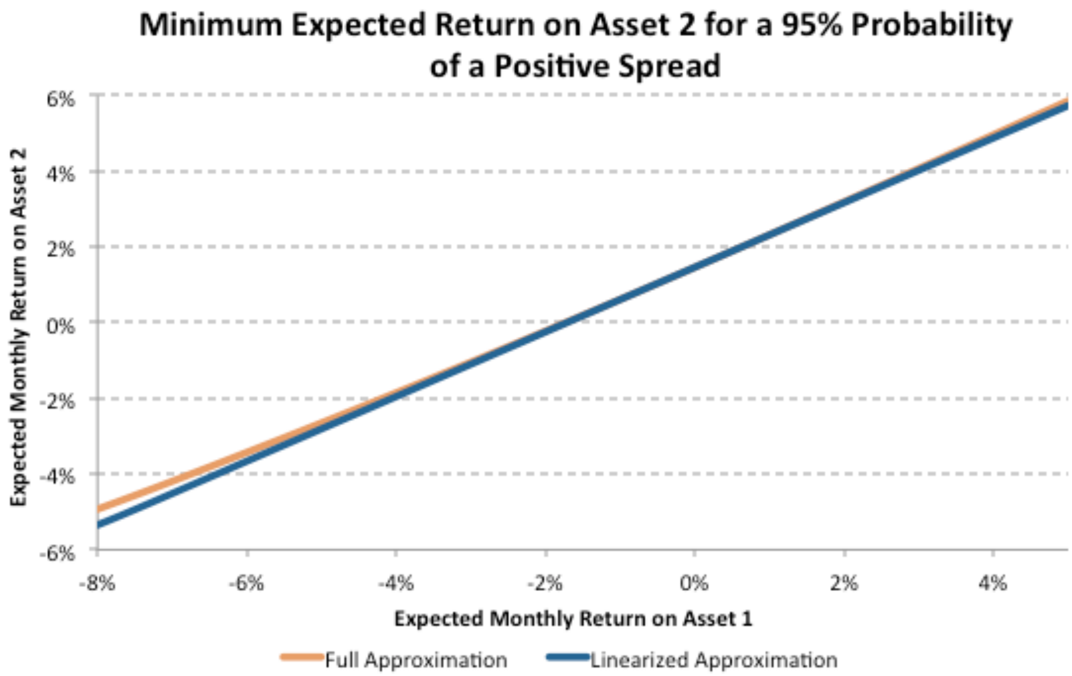
Correlation	-50%	**Time (T-t)**	0.25			0.5			0.75		
		Profit, p	10%	20%	30%	10%	20%	30%	10%	20%	30%
10%	10%	**Intercept**	5.06%	5.63%	6.12%	3.45%	3.74%	3.99%	2.77%	2.97%	3.14%
		Slope	0.87	0.88	0.89	0.87	0.88	0.89	0.88	0.88	0.89
10%	20%	**Intercept**	7.74%	8.30%	8.79%	5.39%	5.67%	5.92%	4.37%	4.57%	4.73%
		Slope	0.87	0.88	0.89	0.87	0.88	0.89	0.87	0.88	0.89
20%	10%	**Intercept**	7.08%	7.67%	8.18%	4.84%	5.15%	5.42%	3.89%	4.10%	4.28%
		Slope	0.88	0.89	0.90	0.89	0.90	0.90	0.89	0.90	0.91
20%	20%	**Intercept**	9.52%	10.10%	10.61%	6.61%	6.91%	7.17%	5.35%	5.55%	5.73%
		Slope	0.88	0.89	0.89	0.88	0.89	0.90	0.89	0.90	0.90

Correlation	0%	Time (T-t)	0.25			0.5			0.75		
		Profit, p	10%	20%	30%	10%	20%	30%	10%	20%	30%
10%	10%	**Intercept**	4.26%	4.81%	5.30%	2.88%	3.16%	3.41%	2.31%	2.50%	2.66%
		Slope	0.87	0.88	0.89	0.87	0.88	0.89	0.87	0.88	0.89
10%	20%	**Intercept**	6.74%	7.29%	7.77%	4.68%	4.95%	5.20%	3.79%	3.98%	4.14%
		Slope	0.87	0.88	0.88	0.87	0.88	0.89	0.87	0.88	0.89
20%	10%	**Intercept**	6.00%	6.58%	7.09%	4.08%	4.38%	4.65%	3.26%	3.47%	3.65%
		Slope	0.88	0.89	0.90	0.88	0.89	0.90	0.89	0.90	0.91
20%	20%	**Intercept**	7.90%	8.48%	8.98%	5.46%	5.76%	6.02%	4.42%	4.62%	4.79%
		Slope	0.87	0.88	0.89	0.88	0.89	0.90	0.88	0.89	0.90

Correlation	50%	Time (T-t)	0.25			0.5			0.75		
		Profit, p	10%	20%	30%	10%	20%	30%	10%	20%	30%
10%	10%	**Intercept**	3.21%	3.76%	4.24%	2.14%	2.42%	2.66%	1.70%	1.89%	2.05%
		Slope	0.87	0.88	0.88	0.87	0.88	0.89	0.87	0.88	0.89
10%	20%	**Intercept**	5.53%	6.07%	6.54%	3.82%	4.09%	4.32%	3.10%	3.27%	3.43%
		Slope	0.86	0.87	0.88	0.86	0.87	0.88	0.86	0.87	0.88
20%	10%	**Intercept**	4.64%	5.22%	5.73%	3.12%	3.42%	3.68%	2.48%	2.68%	2.86%
		Slope	0.88	0.89	0.89	0.88	0.89	0.90	0.89	0.90	0.90
20%	20%	**Intercept**	5.80%	6.36%	6.85%	3.98%	4.26%	4.51%	3.21%	3.40%	3.57%
		Slope	0.87	0.88	0.89	0.87	0.88	0.89	0.87	0.88	0.89

Correlation	90%	Time (T-t)	0.25			0.5			0.75		
		Profit, p	10%	20%	30%	10%	20%	30%	10%	20%	30%
10%	10%	**Intercept**	1.83%	2.37%	2.84%	1.17%	1.43%	1.67%	0.91%	1.09%	1.24%
		Slope	0.86	0.87	0.88	0.86	0.87	0.88	0.86	0.87	0.88
10%	20%	**Intercept**	4.27%	4.79%	5.25%	2.93%	3.19%	3.41%	2.37%	2.54%	2.69%
		Slope	0.85	0.86	0.87	0.85	0.86	0.87	0.85	0.86	0.87
20%	10%	**Intercept**	3.08%	3.66%	4.18%	2.02%	2.32%	2.58%	1.58%	1.79%	1.97%
		Slope	0.88	0.89	0.90	0.89	0.89	0.90	0.89	0.90	0.91
20%	20%	**Intercept**	3.05%	3.58%	4.05%	2.04%	2.30%	2.53%	1.62%	1.80%	1.95%
		Slope	0.86	0.87	0.88	0.86	0.87	0.88	0.86	0.87	0.88

As stated previously, linearization does introduce some error, but it is generally tolerable for our purposes. The following graph shows the deviation of the linearized version for $T = 0.75, p = 0.2, \sigma_1 = 0.1, \sigma_2 = 0.2$, and $\rho = 0.5$.



How Many Birds Did We End Up With?

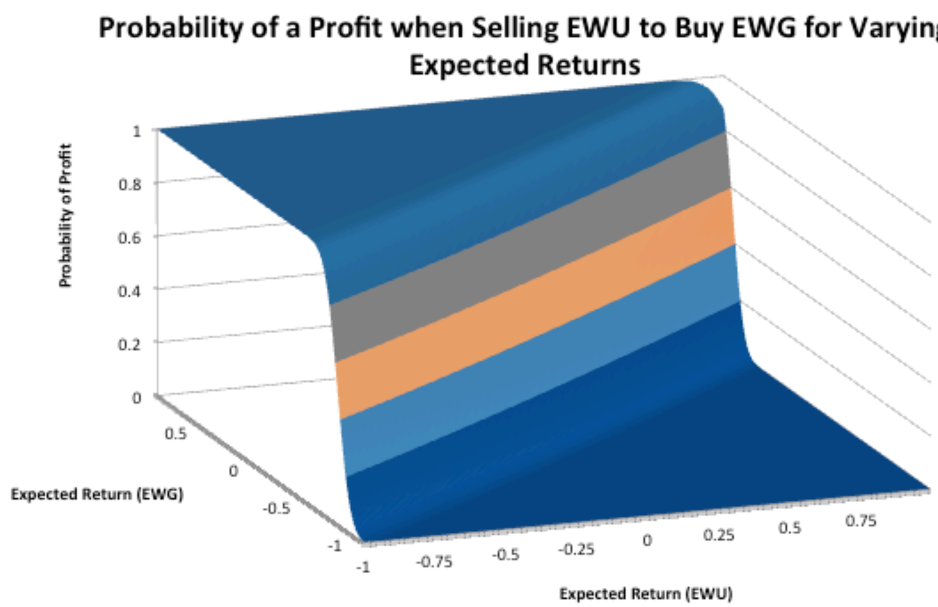
The tradeoff between trading now and paying higher taxes versus possibly missing an investment opportunity in order to get a more favorable tax treatment is not entirely straightforward, and we have shown some ways to determine the "better" option. At tax maturity and in the period before, we can use each bit of newfound hindsight to see how well our methods performed.

How did the probability of switching change over the time period as the parameters changed? Did the signal from our tactical model change? Is there a new tax-tactical opportunity? Did our views on the expected returns of the assets pan out?

At tax maturity, our analysis assumed that asset 1 is sold but asset 2 is kept. If asset 1 has a buy signal at that time, you can hold it and pay the long-term gains when it is sold. If the switch was made, to sell asset 2, the analysis should be repeated.

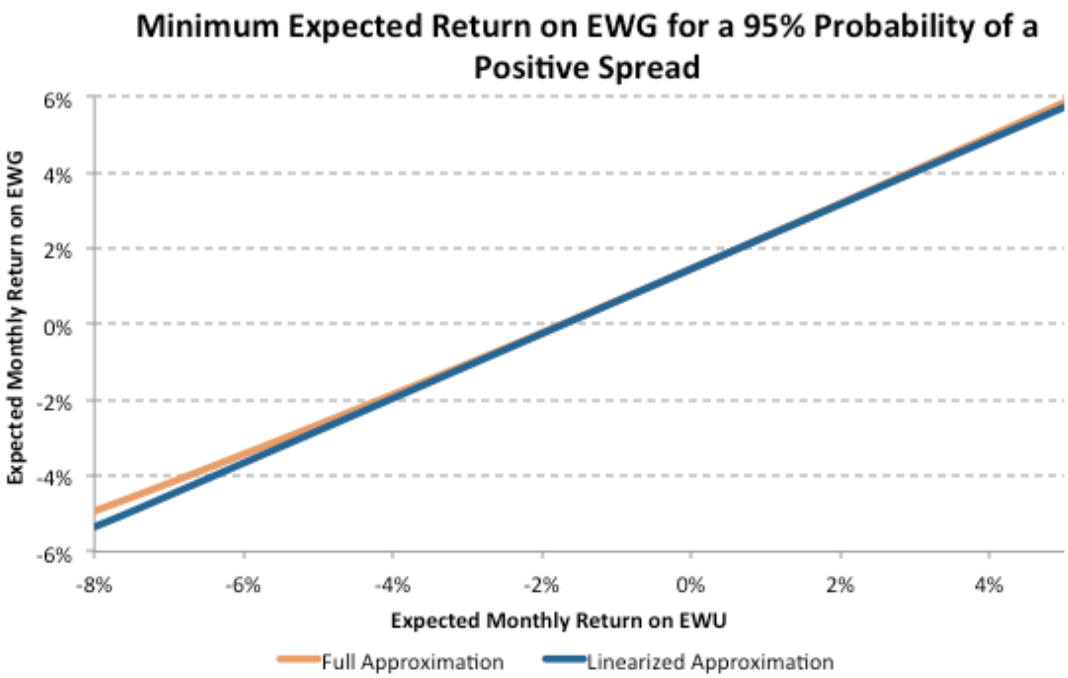
Case Study: When to Buy

As an example, assume that it is September 18, 2012 and we have been holding the iShares MSCI United Kingdom Index (EWU, $\sigma_1 = 9\%$) since June 18, 2012 with a 9.3% profit so far. Our model has just signaled to sell EWU in favor of the iShares MSCI Germany Index (EWG, $\sigma_2 = 13\%$). The historical realized correlation, which we will use to estimate the forward correlation, between the two assets is 84%. The following chart shows the probability for a range of expected annualized returns for the two ETFs.



From the graph, we can see that the probability is the most interesting in the region where the expected returns are somewhat similar. Otherwise, the decision is more clear-cut.

Our current belief for monthly returns on the assets is 0% and 1% for EWU and EWG, respectively. Using our approximations, we can see where the 95% probability cutoff lies for these two assets.



For a 0% belief on EWU, we require 1.5% for EWG; alternatively, for a 1% belief on EWG, we require a belief of *less* than -0.5% on EWU.[10] Since our beliefs fall outside of these thresholds, we decide to wait. In fact, the probability with our views is 85%.

On December 3, 2012, we get another buy signal for EWG. This time, we have $\sigma_1 = 9\%$, $\sigma_2 = 11\%$, $\rho = 92\%$, and $p = 8.5\%$, and our outlook for the assets have changed over the past 3 months. We now believe that EWU will return 0.5% per month while EWG will return 2% monthly over the time remaining until tax maturity. Based on our belief for EWU, EWG's expected monthly return must be above 1.6% to reach the 95% threshold for a positive spread.[11] Since we believe that EWG's return will be greater than this cutoff, we decide to make the trade.

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[10] As a rough guess, we could have interpolated using the tables shown earlier. The intercepts for $\sigma_1 = 10\%$, $T = 0.75$, $\rho = 90\%$, and $p = 10\%$ are 0.9% and 2.4% for $\sigma_2 = 10\%$ and $\sigma_2 = 20\%$, respectively. Linearly interpolating to $\sigma_2 = 12.5\%$ yields a threshold return of 1.3%.

[11] As before, we can use the tables to get a coarse estimate of $1.2\% + 0.86 \cdot 0.5\% = 1.6\%$ as the hurdle that our belief must surpass.

At tax maturity on June 19, 2013, our spread is equal to 5.4%, so in hindsight, the decision to make the trade worked out. However, if we had made the trade when we first analyzed the option, our spread would be 7.6%. This illustrates the important point that the beliefs used as inputs to the method are the most critical piece in making the decision; the method is merely a tool to translate these beliefs into action.

Conclusion

A pillar of outcome oriented investing is providing tactical investment strategies that meet investors' goals. Simply beating a benchmark may not yield the best final result for an investor if the investment is not tax-efficient. Even if a quantitative model signals when an opportunity arises to adjust the allocation, the final allocation decision relies on a more comprehensive model that includes tax tacticallity.

In this paper, we have outlined method for calculating the probability of realizing a positive profit from an allocation change. We laid the foundations by examining the return needed to overcome the hurdle between short and long-tem capital gains tax rates over different time periods.

We then shifted this framework to a probabilistic setting by accounting for volatility in the asset price and developed an equation to calculate the minimum expected annual return to be 95% confident that waiting was more profitable than selling.

We extended the method to include a switch between two securities and derived a way to approximate the likelihood of a positive outcome. By examining the sensitivities to the different input parameters, we provided some intuition for the two asset case and developed heuristics for deciding between similar investment options in a tactical tax framework.

Finally, we further simplified the approximation to a linear equation that can be used to quickly judge whether the 95% probability threshold has been reached based on updated beliefs. This method of calculating the probability of a positive payout can be used with one's market views in order to make an informed, quantitative decision on how to improve tactical asset allocation strategies in a tax conscious framework.

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