Constructing Portfolios of Constant-Maturity Fixed-Income ETFs

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December 2013

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Abstract

In this paper we address the differences in managing risk for constant-maturity fixed-income indices and traditional fixed-income portfolios. Traditional risk measures include *duration*, however the consistent turn over and re-investment nature of constant-maturity indices creates a complicated relationship between yield and duration. We derive that Sharpe optimal portfolios can be found based on a simple yield-to-risk framework where risk is quantified as the volatility of the underlying driving interest rate factor scaled by duration.

Introduction

The proliferation of ETFs has expanded the palette of investible securities for retail investors, especially in fixed-income where investors can access fixed-income sectors including domestic and international government bonds, mortgage-backed securities, corporate bonds, inflation-linked bonds and high yield. With present fears that a 30-year bull market in fixed-income is rapidly coming to a close, most investors have become acquainted with the notion of *duration:* a measure of the percentage decrease in a bond's value for a 100 basis point rise in interest rates. However, constant-maturity fixed-income ETFs are unique in that they constantly turn over their underlying holdings and re-purchase new on-the-run securities, creating a complicated relationship between yield and duration that defines the ETF's sensitivity to changes in interest rates. In this paper, we explore this relationship and derive the *yield-to-risk* measure that we believe allows for the construction of Sharpe optimal portfolios with these ETFs.

A Simplified Constant-Maturity Index

To develop a simple heuristic, we will utilize a simplified constant-maturity index construction methodology. Usually, the basic methodology for a constant maturity index works like so:

1. Purchase the current on-the-run bond
2. While holding the bond, re-invest any coupons distributed
3. Sell the bond and purchase the new on-the-run bond

In practice, many constant maturity indices turn over every month and purchase bonds with semi-annual coupons, so coupons are rarely collected and re-invested.

Our methodology will be simpler: we purchase our bond at the beginning of the period and receive our coupon at the end of the period, at which time we sell our bond and purchase the new on-the-run bond. We also assume that any changes to the prevailing yield occur at the end of the period.

Throughout this paper, we use the notation $B_{i,j}$ to denote the price of a bond at time j that was purchased at time i.

For simplicity in this paper, we will assume that all bonds are bought at par, where par equals $1:

$$B_{t,t} = \$1$$

By letting the bond equal $1 at point of purchase, we can treat yield (written herein as y_t) and coupon as equivalents. This simplified construct allows us to avoid mid-period pricing issues such as sensitivity to time-to-maturity that will occur when yield-to-maturity no longer equals the coupon rate.

Now let us define our wealth (or, equivalently, constant-maturity index) process:

$$w_t = w_{t-1} - n_{t-1}B_{t-1,t-1} + n_{t-1}y_{t-1}\Delta t + n_{t-1}B_{t-1,t}$$

where n_{t-1} is the number of bonds bought at time t-1, y_{t-1} is the yield at time t-1 and Δt is the time in years between t-1 and t. In this expression, the first term is the wealth at t-1, the second term is the total purchase price for bonds at t-1, the third term is the interest income earned during the period and the fourth term is the ending value of the bonds bought at t-1.

If we assume we fully invest our wealth at each step, we can rewrite our number of shares as:

$$n_t = \frac{w_t}{B_{t,t}}$$

Which allows us to re-write our change in wealth as:

$$\Delta w_t = -w_{t-1} + w_{t-1}y_{t-1}\Delta t + w_{t-1}B_{t-1,t}$$

If we define the price of a bond as a function of yield and time, we can use Taylor's theorem to approximate the change in price as:

$$\Delta P = \frac{\delta P}{\delta t}\Delta t + \frac{\delta P}{\delta y}\Delta y + \frac{1}{2}\frac{\delta^2 P}{\delta y^2}(\Delta y)^2 + \cdots$$

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Since we are assuming that we purchased the bond at par, and therefore yield-to-maturity is equal to the coupon rate, we know that $\frac{\delta P}{\delta t} = 0$[1]. Therefore, for small changes in yield, we can approximate the change as:

$$\Delta P \approx \frac{\delta P}{\delta y} \Delta y$$

Dividing both sides by price, we get the familiar derivation of *modified duration* (D):

$$\frac{\Delta P}{P} \approx \frac{1}{P} \frac{\delta P}{\delta y} \Delta y = -D \Delta y$$

This is the familiar derivation of bond *duration* (D), which allows us to re-write (again, with the convenience of assuming that $B_{t,t} = P = 1$):

$$B_{t,t+1} = 1 - D \Delta y_{t+1}$$

Therefore, our above change in wealth is:

$$\Delta w_t = -w_{t-1} + w_{t-1} y_{t-1} \Delta t + w_{t-1}(1 - D \times (y_t - y_{t-1}))$$

Which reduces to,

$$\Delta w_t = w_{t-1}(y_{t-1} \Delta t - D \times (y_t - y_{t-1}))$$

Constructing a Portfolio

Let us first assume that y_t is a random walk with zero drift (i.e. $\Delta y_{t+\Delta t} \sim N(0, \sigma_y^2 \Delta t)$); we then know that:

$$\frac{\Delta w_{t+\Delta t}}{w_t} \sim N(y_t \Delta t, D^2 \sigma_y^2 \Delta t)$$

[1] See Appendix A for derivation

We are provided with a fairly intuitive interpretation of volatility in our wealth process: it is a direct function of the duration of the bonds we are purchasing and the volatility of the underlying yield.

Up until this point, we've spoken explicitly about changes in yield. Let us now consider a portfolio of constant maturity bond indices (including nominal of all durations, credit-based and inflation-linked): how would we construct the maximum Sharpe ratio portfolio? Let us make a simplifying assumption that all yields are simply a fixed spread level, s, above a singular driving interest rate (note that this is not a totally outrageous assumption, as *level* drives a significant portion of yield-curve changes and over short enough periods we can likely claim *ceteris paribus*). To solve for the maximum Sharpe ratio portfolio we would be looking to solve[2]:

$$\max_{\vec{w}} \frac{\vec{w}^T(r+\vec{s})}{\sqrt{\vec{w}^T(\sigma_y{}^2\vec{D}\vec{D}^T)\vec{w}}}$$

Subject to:

$$\vec{w}^T\vec{w} = 1$$
$$\forall w_i \geq 0$$

Since, in this toy example, the constant-maturity indices are all driven by the same interest rate, they are going to be perfectly correlated. Therefore, the simple solution is to invest 100% of your assets in the security with the maximum yield-to-risk, where risk is measured as duration-scaled-interest-volatility (or, as we saw above, volatility of the index's linear returns).

Obviously our toy example is too simple. But if we define $\sigma_w = D\sigma_y$, we can generalize our max Sharpe ratio problem to:

$$\max_{\vec{w}} \frac{\vec{w}^T\vec{y}}{\vec{w}^T\Sigma_w\vec{w}}$$

[2] Assuming 0% risk-free rate

Subject to:

$$\vec{w}^T\vec{w} = 1$$
$$\forall w_i \geq 0$$

In other words, our optimal Sharpe ratio is our optimal yield-to-risk portfolio.

Taking a Simplified View on Rate Changes

What happens if we do have a view on where rates are going? Consider defining y_t as a random walk with drift μ_y (implying $\Delta y_{t+\Delta t} \sim N(\mu_y \Delta t, \sigma_y^2 \Delta t)$). Our change in wealth then becomes:

$$\frac{\Delta w_{t+\Delta t}}{w_t} \sim N((y_t - D\mu_y)\Delta t, D^2\sigma_y^2 \Delta t)$$

What we find is that our yield is simply shifted by our duration-scaled drift factor; i.e. how much of the yield will be left after our expected bond price loss is accounted for. Our solution for our maximum Sharpe portfolio remains much the same, except we define $\tilde{y} = y - D\mu_y$.

A More Realistic Rate Model

Let us assume a more complicated model for y_t; in particular, let us assume that y_t is a discrete-time Vasicek model:

$$y_{t+\Delta t} = y_t + a(b - y_t)\Delta t + \sigma_r \sqrt{\Delta t} Z_{t+\Delta t}$$

Substituting, we find that:

$$\frac{\Delta w_{t+\Delta t}}{w_t} \sim N((y_t(1 + aD) - abD)\Delta t, D^2\sigma_y^2 \Delta t)$$

Once again we find the solution to our maximum Sharpe ratio portfolio will be one relying on yield and volatility; this time, our yield will be scaled and differenced by our expectations for how quickly, “a”, it will revert back to its long-term mean level, “b”.

Theory & Reality

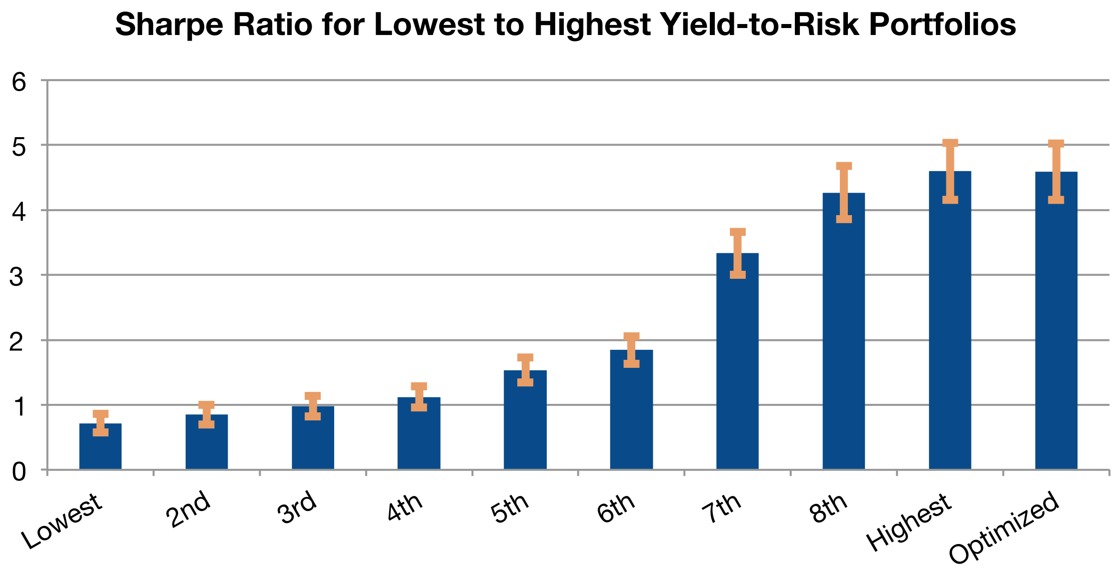
In our toy example, we've derived that yield-to-risk can lead to the Sharpe optimal portfolio; but reality is far messier than theory. How well does our toy translate to reality?

To test, we used daily interest rates (from the St. Louis Federal Reserve's Economic Data ("FRED") platform) from 1993 for 3-month, 6-month, 1 Year, 2 Year, 3 Year, 5 Year, 7 Year, 10 Year and 20 Year Treasury bonds. The rates were then transformed into constant-maturity price indices, assuming semi-annual coupons and monthly turnover (with the exception of 3-month rates, which assume quarterly coupons).

We then construct nine portfolios with increasing yield-to-risk ratios. The first portfolio will always have the lowest yield-to-risk ratio while the ninth portfolio will always have the highest yield-to-risk ratio. Portfolio constituents are rebalanced on a monthly basis so that we allow enough time for the factor to express itself. To avoid sampling bias, each index is broken into four equal buckets. On the first week, the first bucket is allocated with respect to that week's yield-to-risk rankings; on the second week, the second bucket is allocated with respect to that week's yield-to-risk rankings; et cetera. On the fifth week, the cycle restarts with the first bucket. By using this methodology, we avoid potential sampling bias by sampling at multiple points each month while also maintaining the desired holding period of a month.

We also construct an "optimized" portfolio that creates a maximum yield-to-risk portfolio utilizing all available underlying indices. Furthermore, the optimization process takes advantage of covariance information, recognizing that rate changes along the curve are not all purely linear shifts, but also changes as functions of slope and curvature.

The results follow:



Testing with ETFs

In a second test, we utilize various fixed-income ETFs. The first major difference is that instead of utilizing known forward rates, we approximate yield with exponentially weighted trailing 252-day dividend yield. The second major difference is the impact of correlations on portfolio construction, which we will touch upon later. The ETFs utilized are: SHY, IEI, IEF, TLH, TLT, CSJ, LQD, SHYG, HYG, STIP and TIP. As the ETF data becomes available, it is utilized in the construction of the portfolios.

In our toy model, we assume yield and interest rates are interchangeable, and therefore, in the short-term, duration risk is the only risk that drives changes in bond prices. In reality, credit and inflation spreads are significant driving factors in credit-based and inflation-linked bond prices. Therefore, an underlying assumption in our model is that duration is replaced by a general linear factor that captures the impact on bond price due to a change in yield, whether that change in yield is due to rate changes, credit-spread changes, or inflation-spread changes. We also assume that the dividends paid by these ETFs are a valid proxy for the yields from the underlying securities.

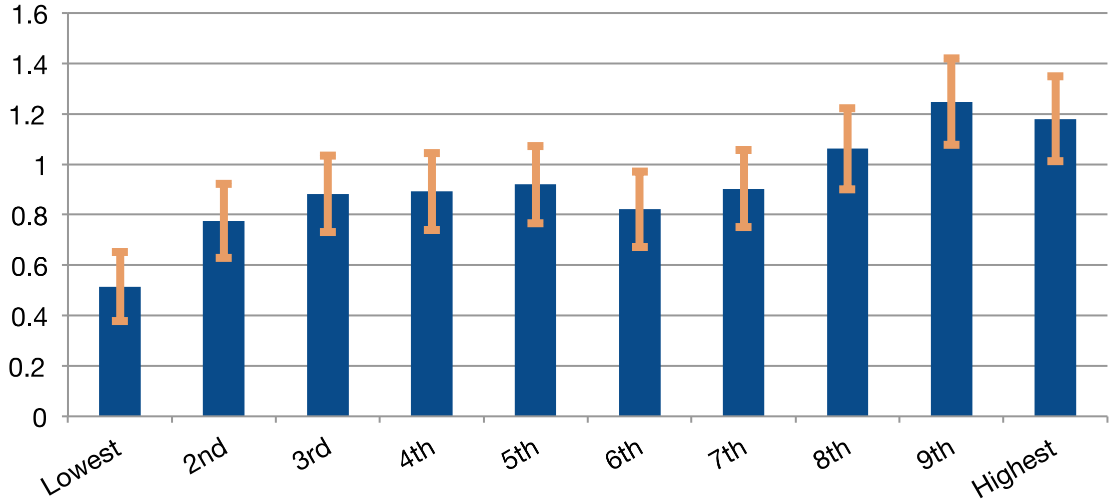
Furthermore, many of these ETFs are not explicitly constant maturity (e.g. LQD and HYG) and their underlying securities are not necessarily consistent throughout time. For example, whereas a 7-10 year constant-maturity Treasury index will always sell U.S. Treasuries to purchase U.S. Treasuries, the Markit iBoxx USD Liquid High Yield Index may sell a basket of bonds from one group of issuers and purchase a basket of bonds from a completely separate group of issuers. In our model we gloss over these details and assume that diversification allows us to treat these baskets as if they were a singular, summary bond representing the entire market and that the maturity of this bond is equivalent to the weighted average maturity of the index (also, therefore, assuming that the weighted average maturity remains fairly consistent throughout time).

The test process had to change due to the high impact that correlations will have on the portfolio construction process with these ETFs. By only sorting based on individual yield-to-risk, we could theoretically create a low yield-to-risk portfolio from two high yield-to-risk securities with negative correlation to each other but label it as high yield-to-risk. Therefore, any naïve combination of individual securities may lead to incorrect quantile rankings. Furthermore, a naïve ranking process may lead to quantiles that are sensitive to different risk factors, and therefore the results would be highly path dependent upon which risks were realized during the backtest period.

The test was designed such that at each point in time, ten portfolios are constructed, each with a yield target a fixed excess level above the current minimum yield offered. The target yield levels are equally spaced between the current minimum yield offered and the current maximum yield offered and each portfolio seeks to match this yield target with the minimum volatility level possible. Ten quantile-based portfolios are then constructed based on the yield-to-risk of the target excess yield portfolios.

The results follow:

Sharpe Ratio for Lowest to Highest Yield-to-Risk Portfolios



While the results are not as clear-cut as the rate-based constant-maturity index test, there is a significant (5% level) increase in Sharpe ratio between higher yield-to-risk portfolios versus lower yield-to-risk portfolios.

Conclusion

With the fear of rising interest rates, many investors have become familiar with *duration* risk. However, risk management of constant maturity fixed-income indices is more complex than simply minimizing duration exposure. Constant-maturity fixed-income portfolios are unique in that they frequently turn over their holdings, reinvesting capital and coupons into new on-the-run bonds, creating a complicated relationship between yield and duration.

In this paper we have derived that subject to certain simplifying assumptions the Sharpe optimal portfolios will maximize their yield-to-risk ratio, where risk is measured by the volatility of the underlying driving interest-rate factor scaled by duration – or, equivalently, the volatility of the constant maturity index.

We then derive different manipulations to the yield-to-risk framework that can be made based on underlying assumptions and beliefs of the driving interest rate factor.

Finally, we explore whether the toy model derived in this paper holds for live market data, from rate-based constant-maturity indices to ETFs. We find that our results are significant at a 5% level for both rate-based constant-maturity indices and portfolios of ETFs.

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Appendix A

The basic bond value equation is:

$$B(T,r) = \frac{C}{r} \times \left(1 - \frac{1}{(1+r)^T}\right) + \frac{P}{(1+r)^T}$$

Where P is the principal, C is the coupon, T is the time-to-maturity and r is the discount rate yield-to-maturity. C is also defined as $y \times P$, where y is the coupon interest rate.

Taking the partial derivative with respect to T, we get:

$$\frac{dB}{dT} = -\frac{C}{r}\frac{1}{(1+r)^T}\ln(1+r) + \frac{P}{(1+r)^T}\ln(1+r)$$

Purchasing the bond at par means that $y = r$, which reduces the equation to:

$$\frac{dB}{dT} = -\frac{P}{(1+r)^T}\ln(1+r) + \frac{P}{(1+r)^T}\ln(1+r) = 0$$

When a bond is trading at par $y = r$ and therefore $\frac{dB}{dT} = 0$. Note that this relationship only holds from one coupon payment date to the following coupon payment date due to accrued interest.

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